MATH 191 —RAMSEY THEORY

Basic Information

Instructor: Nick Ramsey

Course objectives: A classic theorem of Ramsey states that for any n, there is a k such that a graph of size k must contain either a complete or empty graph of size n. Informally, this suggests that perfect chaos is impossible: inside a graph of sufficiently large size, there must be a large subset which looks very uniform. Later developments in many areas of mathematics—including set theory, additive combinatorics, dynamics, and computer science—offered variations on this theme, producing a body of loosely related results known under the banner 'Ramsey Theory.' In this course, we will begin by proving Ramsey's theorem and sketching some of its applications. Then we will survey the various different Ramsey-type theorems that have appeared in these different areas. The aim is to give exposure to the way Ramsey-theoretic ideas enter into several different kinds of mathematics. No prior experience with graphs or logic is required and all of the topological, logical, and combinatorial notions needed will be introduced and developed along the way.

Textbook: There is no required text for the class, though the material for the course will be drawn largely from the references listed in the course outline below.

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Grading: Grades for this course will be computed according to the following two distribution:

- Homework: 50%
- Final presentation: 20%
- Final: 30%

Grading is not done on a curve, but grades are instead assigned based on whether or not students have successfully mastered the material of the course.

Homework: Homework problems will be posted on the CCLE page for each week as the semester progresses. The lowest score will be dropped.

Exam and Presentation: The final exam will be a take-home final released on the last day of class and due the last day of finals week. The last week of class will be dedicated to in-class presentations. In week 6, a list of possible topics for the final presentations will be circulated and students will be asked to schedule a time to meet with the instructor by the end of week 7 to discuss their topics and the details of the presentation.

Course Outline: The plan is to cover the following topics, spending roughly 2 weeks on each:

(1) Ramsey's theorem; its proof and applications.
  - Ronald L. Graham, Bruce L. Rothschild, and Joel H. Spencer. *Ramsey theory.* Vol. 20. John Wiley & Sons, 1990.
  - Matthew Katz and Jan Reimann. *An Introduction to Ramsey Theory.* Vol. 87. American Mathematical Soc., 2018.

(2) Ellis's Lemma, Hindman's Theorem, and the Hales-Jewett Theorem.
  - Mauro Di Nasso, Isaac Goldbring, and Martino Lupini, *Nonstandard methods in Ramsey theory and combinatorial number theory*, Lecture Notes in Mathematics, Volume 2239 (2019).

(3) Extensions of Ramsey's theorem to uncountable sets: Sierpinski's partition, the Erdős-Dushnik-Miller Theorem, the Erdős-Rado theorem.
  - Paul Erdős, et al. *Combinatorial set theory: partition relations for cardinals.* Elsevier, 2011.
  - Akihiro Kanamori. *The Higher Infinite: large cardinals in set theory from their beginnings.* Springer Science & Business Media, 2008.

(4) Topological Ramsey Theory: the Galvin-Prikry Theorem and the Ellentuck topology.
  - Stevo Todorcevic. *Introduction to Ramsey spaces.* Vol. 174. Princeton University Press, 2010.

(5) Structural Ramsey Theory: Fraïssé's theorem and the Kechris-Pestov-Todorcevic Correspondence.
  - Manuel Bodirsky. "Ramsey classes: examples and constructions." *Surveys in combinatorics* 424 (2015): 1.
  - Peter J. Cameron, *Oligomorphic Permutation Groups.* London Mathematical Society Lecture Note Series, Series Number 152, 1990.
  - Wilfrid Hodges, *Model theory.* Cambridge University Press, 1993.
  - Lionel Nguyen van Thé. "A survey on structural Ramsey theory and topological dynamics with the Kechris-Pestov-Todorcevic correspondence in mind." *arXiv preprint* arXiv:1412.3254 (2014).

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